A STUDY OF DIFFRACTION

PHYSICS 410/510

EXPERIMENT O-3

(revised 5/17/95 by T. A. Cool)

Abstract

The Kirchhoff scalar diffraction theory provides simple and useful analytical solutions to several practical problems in the diffraction of light. Among these are the diffraction of light passing through circular aperture and rectangular slits, and through multiple parallel slits (*e.g.*, transmission gratings). The diffraction of light past an edge and around an opaque circular disk, and the intensity distribution in the focal region of a spherical lens are three additional examples for which simple analytical solutions are available.

Intensity distributions for each of these important examples may be recorded and compared with calculations in this experiment. The student will gain familiarity with several topics in physical optics including the Cornu spiral, Arago's spot, Babinet's principle, the Airy disk, Fresnel and Fraunhofer diffraction, and the Rayleigh criterion for spatial resolution.

I. Experimental Apparatus

A. Hardware

Fig. 1 illustrates the setup available for the experiments. An unfocused and unpolarized 1 mW 633 nm He-Ne laser beam (Spectra-Physics model 155a) passes through a 25 micron diameter spatial filter (Newport, model 900 with 25 micron pinhole, 10x 0.25 objective) to eliminate spurious high spatial frequencies to provide a diffraction-limited light source with a smooth near-Gaussian transverse intensity profile. The laser and spatial filter are mounted on a two-meter optical bar, which accommodates additional optical accessories. These include various diffracting apertures, lenses, a viewing screen, and a 165 x 192 pixel CCD video camera controlled with a laboratory computer (IBM 8086).

B. Software tools

The IBM PC has several software tools that aid in data acquisition and analysis. Two key executable programs are called *metawndo* and *wincam*. *Metawndo* enables the graphics driver and *wincam* displays the CCD camera's image on the monitor screen. For simplicity both of these programs are executed from a batch file called CAMERA.BAT.

To operate the camera, type <CAMERA> from the directory c:\diffract>. This command will place you in the CCD-environment. Once in this environment all operations are menu driven. At the bottom of the screen, the user sees a list of options whose first letters are in parentheses. To choose one of these options, type the first letter. For example, to choose the <M>ENU option, press the 'M' key. This will bring up a list of choices including <E>XPOSURE, <F>ILES, <C>AMERA, etc. The <E>XPOSURE option is used to set the length of time in milliseconds during which the CCD array integrates the charges of the image pixels. The <F>iles option enables an image to be saved to a <filename>.buf file for subsequent analysis. An annoyance, at present, is the fact that the <Q>UIT menu option to leave the camera environment and return to DOS doesn't always work. In this case you must simultaneously type the <Ctrl>, <Alt>, and <Delete> keys to reboot the computer.

While in the CCD environment the first thing you will want to do is set the exposure time. To do so, press 'M' and then 'E'. Exposure times on the order of one second are reasonable. The exposure time and aperture size on the camera should be set to give good contrast without saturation of the image in regions of high intensity. The dynamic range of the CCD array is only 8-bits (256 output steps). Proper recording of both high intensity and low intensity features of the diffraction pattern may require nearly mutually exclusive (but partially overlapping) records of features that differ greatly in intensity.

Once the camera is focused and the exposure time is set you will want to save the image in a <filename>.buf file. To do so, go into MENU-FILE-SAVE. Then type the <filename>; the .BUF extension will be automatically appended.

After saving a diffraction image, type MENU-CAMERA to turn the camera back on. Because the beam intensity has a Gaussian-like transverse intensity distribution, it will be necessary to make corrections to the diffraction image for quantitative comparisons with theories based on the assumption of a uniform transverse beam intensity. For this purpose remove all diffraction obstacles and save an image of the laser intensity distribution. Finally it will be useful to block the laser from the viewing screen and record the "background" response of the CCD array; this record may be used to correct for possible "glitches" in the residual pixel charge pattern.

A second very important software tool is a numerical charge integrator called

linpat. Linpat is used to integrate the charge distribution for each horizontal row of pixels between selected limits. This enables vertical slices of the image to be defined to provide a vertical cross section of the intensity distribution. There are 192 pixels in each horizontal row and 165 pixels in each vertical column. As an example, suppose we are interested in the vertical intensity distribution at the middle of the horizontal arrays. We may often be able to take advantage of the fact that the horizontal gradient in intensity is small enough that the signal-to-noise of our vertical intensity pattern may be reduced by averaging over several pixel columns. We might therefore integrate horizontally between columns located between horizontal coordinates X1 and X2, subject to the constraint: $1 \leq X1 \leq X2 \leq 192$. The vertical distribution might be, for example, a 9-column average with X1=92 and X2=100. (It is worthwhile at this point to mention that the 165 pixel vertical dimension on our computer screen is actually the horizontal dimension on the diffraction viewing screen.) The reason for this is that linpat only processes vertical patterns and, if we desire to record diffraction patterns that vary horizontally (such as the diffraction) from a vertically oriented straight edge), we must turn the camera on its side.

Suppose, for example, we have recorded the diffraction pattern for the straight edge in a file called DIFFRACT.BUF, we have recorded the unblocked beam profile in a file called PROFILE.BUF, and the "zero-intensity" background pixel pattern in a file called BACKGRND.BUF. These three files are in binary form and must be converted to an ASCII number file representing the vertical intensity patterns, averaged between selected horizontal limits. To generate the vertical intensity profile for the diffraction pattern, corrected for the background, type the following:

C:\LINPAT DIFFRACT.BUF BACKGRND.BUF X1 X2 > <OUTFILE1>.TXT

where OUTFILE1.TXT is the desired name of the generated ASCII file.

To next generate the corrected vertical intensity profile for the unblocked beam, type:

C:\LINPAT PROFILE.BUF BACKGRND.BUF X1 X2 > <OUTFILE2>.TXT

These files each consist of a single column of 165 numbers, in a suitable form for analysis in a spreadsheet or graphical analysis package of your choice. Lotus 123 is installed on the laboratory PC in the directory C:\123. The first step in analysis is the conversion of each file into a two-column XY format with X as the pixel number ranging from 1 to 165 and Y the corresponding intensity value. Secondly, correction for the near-Gaussian beam intensity may be made by dividing the Y(i) intensity value of each pixel (i) of OUTFILE1.TXT by the Y(i) intensity value of the corresponding pixel (i) in OUTFILE2.TXT. (These steps are easily carried out in graphical tools such as GENPLOT or PLOT; in spreadsheets such as LOTUS, these two steps are performed in reverse order.)

As an example of the use of LOTUS, suppose we want to generate a two column XY file:

```
cd\123              {LOTUS resides in directory 123}
123                 {to execute LOTUS}
/                   {to get menu}
file                {import Y(i) values from a filename.txt file}
import
numbers
filename
/
data
fill
b1..b165            {create the X-array of numbers from 1 through 165}
begin at 1
step 1
end 165

graph
type
XY
X
B1..B165            {define X-array}
A
A1..A165            {define Y-array}
VIEW                {view XY graph on computer screen}
SAVE                {creates XY-file in binary form designed only for LOTUS}
```

II. Theoretical Background

A brief summary of the key features of the Kirchhoff scalar diffraction theory is presented as an Appendix to these notes. This material has been drawn from the discussions of diffraction presented in three key references:

E. Hecht and A. Zajac, *Optics*, (Addison-Wesley, Massachusetts, 1979)

J. B. Marion, *Classical Electromagnetic Radiation*, (Academic Press, New York, 1965)

M. Born and E. Wolf, *Principles of Optics*, (Pergamon Press, New York, 1970).

The results given in this Appendix are sufficient for our purposes, but the student is urged to take this opportunity to become familiar with the treatments of diffraction in these useful references.

III. Outline of the Experiments to be Performed

A. The Airy Disk

Adjust the position of the laser beam so that it strikes the center of the viewing screen located about 140 cm from the output plane of the laser. Place the spatial filter assembly (dual micrometers, pinhole, objective lens, mounted on Ealing XY translator) on the optical bar so that the pinhole is located about 10 cm from the output plane of the laser. Carefully adjust the horizontal and vertical positions of the spatial filter to obtain the brightest possible diffraction image centered on the viewing screen (this must be done with the room darkened). Observe the central lobe of the diffraction pattern (Airy Disk) and the weak concentric rings, which surround it. At this point we will postpone recording this diffraction pattern with the CCD camera until we become more familiar with the operation of the camera.

B. Edge Diffraction Problem

Place a razor blade, with its edge aligned vertically, on the optical bar immediately after the spatial filter. The blade should be positioned so that it intercepts about half of the central lobe of the Airy disk. With the room darkened, observe the diffraction pattern on the viewing screen. Notice the diffraction fringes immediately outside of the geometrical shadow of the razor blade.

Now you are ready to observe the pattern with the CCD camera and make measurements suitable for comparison with the theory outlined in the Appendix.

Step One: Position the camera on its mount so that the knurled brass ring is located about 22 cm from the center of the viewing screen. The camera must be located off-axis to avoid blocking the diffracted beam. Set the camera focus to about 3 meters and the f-stop at the F/22-F/16 position. With the camera operating with an exposure time of about 500 msec, adjust the viewing direction of the camera until the first few interference fringes appear as horizontal bands on the monitor display.

Now increase the exposure time to about 1500 msec and adjust the camera f-stop for best contrast and slightly adjust the focus to sharpen the image as much as possible.

Step Two: Record the diffraction pattern. Two overlapping camera recordings will be necessary to examine the first 15-20 interference fringes. One record should display the first 4 or 5 fringes and the geometrical shadow. Save each recording to files, e.g., Edge1.buf and Edge2.buf; be sure to also record the laser beam intensity profile for each camera position, with the razor blade removed. These profiles should be written to files, e.g., Edge1b.buf and Edge2b.buf. Finally, with the room darkened and the laser blocked, record the pixel background response and write to a file, e.g., Backgrnd.buf.

Step Three: Perform data analysis.

1. Experiment

Begin with linpat; for example:

C:CD\DIFFRACT

C:\DIFFRACT\LINPAT EDGE1.BUF BACKGRND.BUF 80 120 > EDGE1.TXT

C:\DIFFRACT\LINPAT EDGE2.BUF BACKGRND.BUF 80 120 > EDGE2.TXT

C:\DIFFRACT\LINPAT EDGE1B.BUF BACKGRND.BUF 80 120 > EDGE1S.TXT

C:\DIFFRACT\LINPAT EDGE2B.BUF BACKGRND.BUF 80 120 > EDGE2S.TXT

We now have four ASCII single column numeric files. At this point use your favorite spreadsheet or graphical analysis package to convert each file to XY format; that is, X_i is the x-coordinate ranging from 1 to 165 for the 165 corresponding values of Y_i. Finally factor out the beam intensity profile by computing for each X_i the ratio $Y_i(edge1.txt)/Y_i(edge1s.txt)$; do the same for the edge2 data. These results may be filed in an ASCII XY numerical format for further analysis (*e.g.*, edge1.dat and edge2.dat). Now by trial and error you may splice together the two overlapping edge1.dat and edge2.dat to create the overall diffraction pattern; this result should be filed in XY numerical format, *e.g.*, diffract.dat.

2. Theory

For comparison with theory you must generate the integrals

$$I(p)/I_0 = 1/2\{[C(x)+1/2]^2 + [S(x)+1/2]^2\}$$

This is easily done in GENPLOT, for example, with the commands:

```
CREATE Y=COS(PI*X*X/2) FROM 0 TO 10 -POINTS 1000
TRANSFORM Y INT
LET Y=Y+1/2
LET Y=Y*Y
ARCH CSQUARE
CREATE Y=SIN(PI*X*X/2) FROM 0 TO 10 -POINTS 1000
TRANSFORM Y INT
LET Y=Y+1/2
LET Y=Y*Y
```

```
ARCH SSQUARE
LET Y=Y+CSQUARE:Y
LET Y=Y/2
WRITE DIFFRACT.THY    (to create an ASCII file for further analysis)
```

To generate the intensity pattern in the geometrical shadow repeat the above with limits: FROM 0 TO -2 -POINTS 1000. Finally append both calculations to get the complete theory ranging from X=-2 to 10.

3. Compare experiment with theory

Normalize the data in diffract.dat so that I/I_0 is asymptotic to 1.0 for large x. Adjust x-range of data to match interference peaks with theory. Figure 2 displays the results of such an analysis; the agreement is remarkably close!

C. Babinet's Principle

The idea here is to examine the intensity patterns for a circular aperture and for a circular disk; a qualitative comparison with the on-axis intensity patterns discussed in the Appendix is desired.

Figure 3 displays the theoretical intensity distribution

$$I(p)/I_0 = \sin^2(ka^2/2R)$$

for the circular aperture. To examine this qualitatively, observe the diffraction pattern on the viewing screen for a 1 mm diameter pinhole aperture placed a few centimeters from the spatial filter. Vary the separation of the aperture from the spatial filter and record the separations that give maxima or minima in the on-axis intensity. Compare your results with the predictions of Figure 3. You may want to record camera images of the intensity patterns for various separations.

Now use a circular dot about 0.5 mm in diameter (use one of the 35 mm slides) to observe Arago's spot and qualitatively investigate the on-axis intensity distribution, predicted (in the Appendix) to be:

$$I(p)/I_0 = A^2R^2/4(R^2+a^2)^2$$

Explain what Babinet's principle is and explain why the intensity distributions are so much different for the complementary geometries. (see Marion, Chpt. 12).

D. The diffraction grating

Obtain camera images for single slit, double slit, and multiple slit diffraction patterns. Use the 35 mm slides for these slits. Analyze your patterns and plot the intensity distributions for each case. You may find it necessary to use a focusing lens to reduce the size of the intensity pattern on the viewing screen. Discuss your results in terms of the theory of a transmission grating with a large number N of slits.

E. Record the Airy Diffraction Pattern

Now that you are experienced with the use of the CCD camera, you may record the Airy diffraction pattern produced by the spatial filter. One problem is that the pattern diverges quite rapidly so that the image is too large on the viewing screen unless the viewing screen is located quite close to the spatial filter. This may be overcome by placing the spatial filter very close to the screen. Because the CCD camera is mounted off-axis, it is possible to place the spatial filter within 10 cm of the viewing screen without obstruction.

A second problem is the fact that the 8-bit dynamic range of the CCD may cause saturation of the peak intensity at the center of the pattern. When the saturation is severe a vertical band may appear on the computer screen; this is an experimental artifact attributable to charge transport ("bleeding") along the vertical columns of pixels. It is possible with careful adjustment of the f-stop setting and using long exposures (1-1.5 s) to minimize this problem. It also helps to use the vertical edge of a small card to block-off half of the Airy pattern (the card is placed just in front of the camera lens).

Figure 4 shows experimental data compared with theory for the Airy pattern. Theoretical curves of I/I_0 and $10 \times I/I_0$ are given (I_0 is defined here to be the on-axis intensity). The data have been normalized to the $10 \times I/I_0$ theoretical curve for comparison. The theoretical curves show that the central disk is much stronger than the first and second rings. The data show clear evidence of saturation of the CCD response; a 10-bit CCD would be useful here. Data of Fig. 4 were taken with much of the central disk obscured with a card placed in front of the camera lens. This provides a better look at the Airy rings.
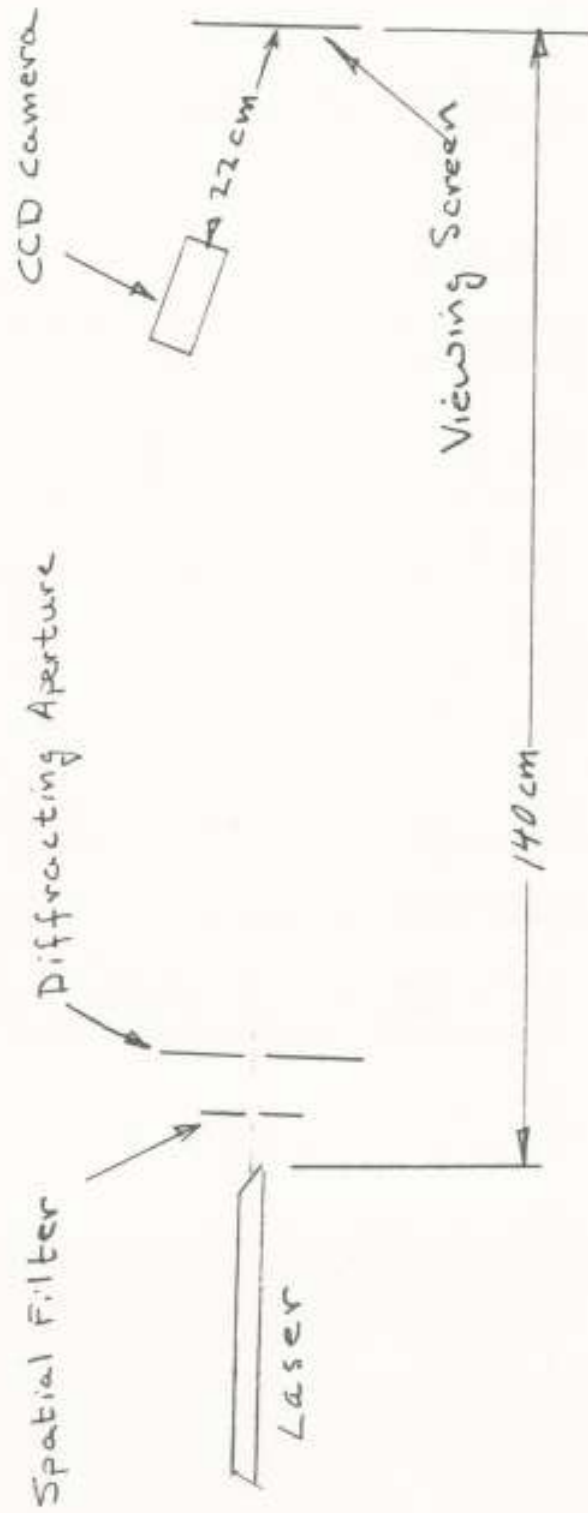
Figure 1: Schematic diagram of apparatus

Figure 2: A comparison of experiment (solid points) and theory (solid line) for the diffraction of a plane wave by an edge.
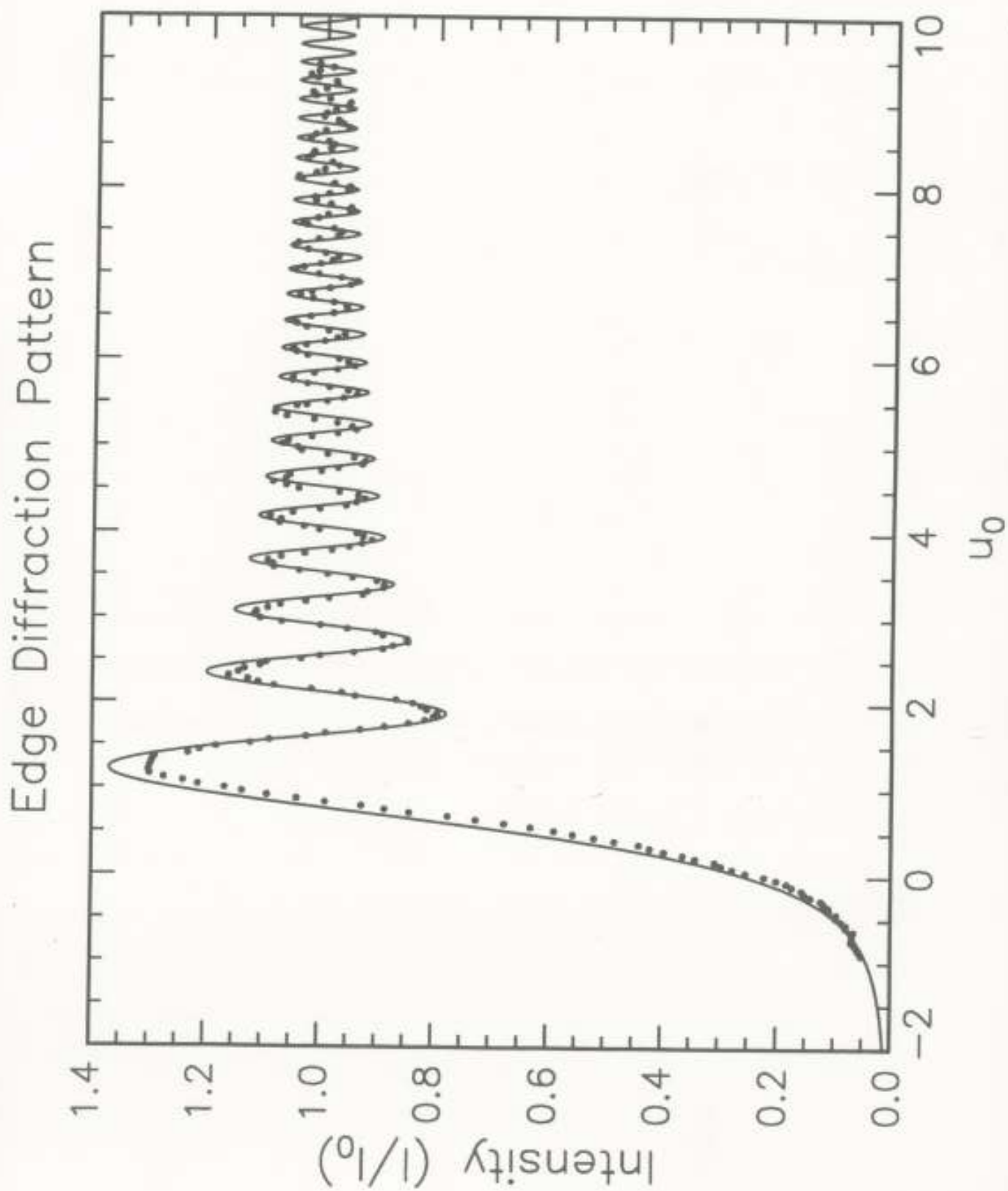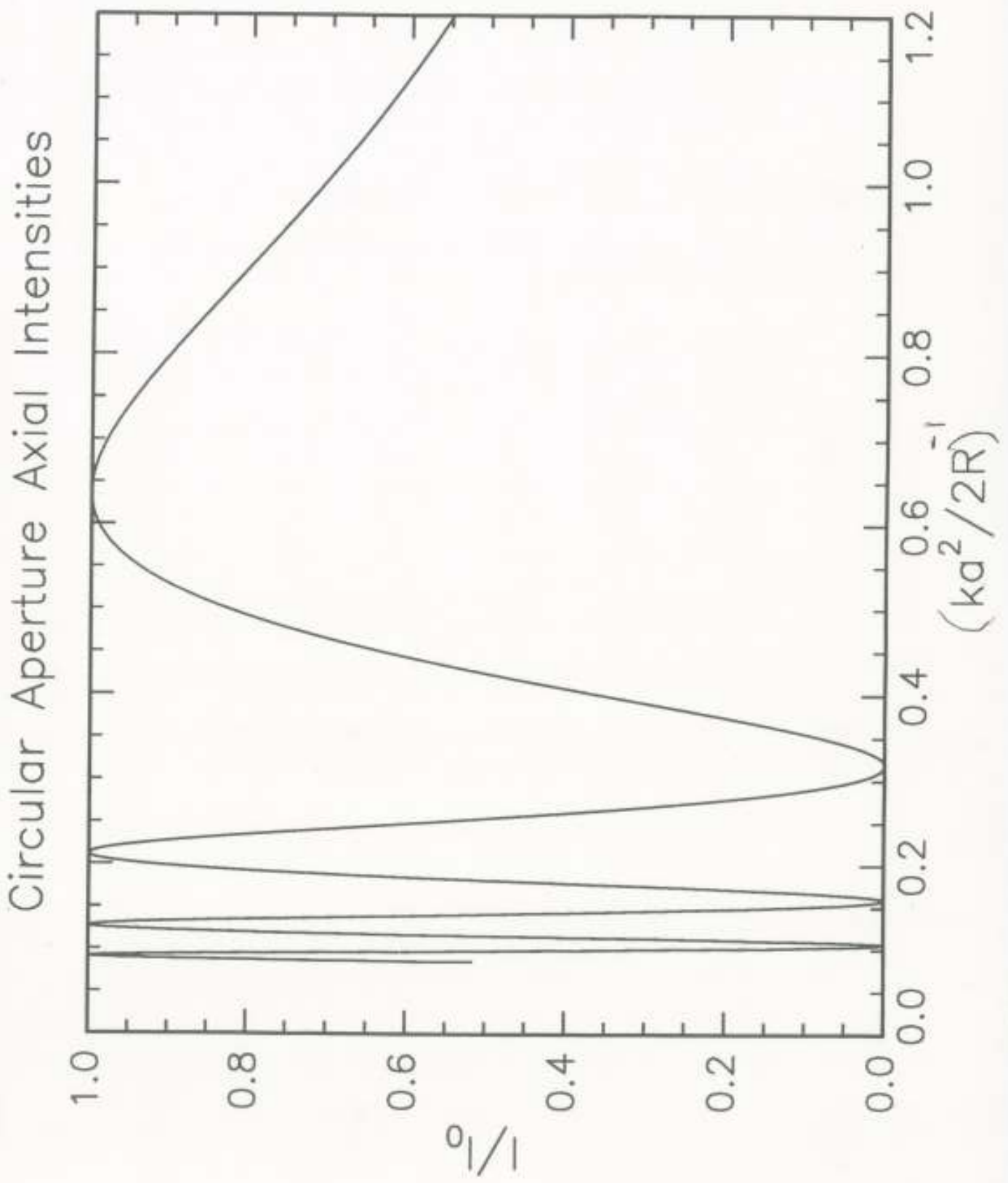
Figure 3: The axial intensity pattern expected for the diffraction of a plane wave by a circular aperture of radius a. R is the distance along the axis from the diffracting aperture and k is the wavenumber ($2\pi/\lambda$).

Figure 4: A comparison of experiment (solid points) and theory (solid lines) for diffraction by a circular pinhole of 25μ diameter (spatial filter). The data are compared with the 10 X expanded scale to examine the Airy rings. The intensities of the Airy rings are too weak to permit accurate simultaneous recording of the Airy disk (central lobe) and the rings; attempts to do this with an 8-bit CCD result in saturation of the Airy disk, as the data clearly indicate.